Cooperation and Competition on Exchange of Network Connectivity Services

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User Provided Networks (UPNs)

- Exploit user idle resources or opportunistic advantages to improve network services to the entire user community.
Handheld $A$ relays Internet traffic of $B$ via an $A - B$ D2D link when:

- $A$ has a good channel and $B$ doesn’t.
- $A$ has excess quota in her plan and $B$ doesn’t.
- $A$ has redundant battery reserves while $B$ doesn’t and prefers to use the low power D2D link.

$A$ will do so anticipating $B$ to reciprocate in the future, or

$A$ will do so anticipating similar service from $C$ who owes to $B$ in reciprocity.
Collaborative Network Services

- A mechanism based on the Nash bargaining solution + virtual currency.

- Users are modeled through payoff functions.
  - Utility from consuming data, *energy cost* and *monetary cost* for serving data, virtual currency benefits.

- Efficiency and Fairness are addressed by the Nash Bargaining Solution.
  - Pareto optimal.
  - Takes into account the standalone operation of each node.
  - Self-enforcing, hence users agree to apply the policy.

- Virtual Currency solves the *double coincidence of needs and wants* problem.

- Decentralized implementation is possible if necessary.
  - Dual decomposition of a convex optimization problem (the NBS problem).

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CoNeS: Collaborative Network Sharing System

- Prototype implementation on Linux handhelds (and later on Android phones).
  - SDN-enhanced mobile devices: implement a programmable packet forwarding datapath.
  - Cloud service: monitors the nodes, and devises the policy.
  - Agnostic to radio interface.
  - Highly adaptive to changing network conditions and user needs.

Directed, time-evolving graph: \( \mathbf{C}(t) = (C_{ij}(t) \in \{0, 1\} : i, j \in \mathcal{N}) \).

Infrastructure access configuration: \( \mathbf{C}_0(t) = (C_{i0}(t) \in \{0, 1\} : i \in \mathcal{N}) \).
Directed, time-evolving graph: $\mathbf{C}(t) = (C_{ij}(t) \in \{0, 1\} : i, j \in \mathcal{N})$.

Infrastructure access configuration: $\mathbf{C}_0(t) = (C_{i0}(t) \in \{0, 1\} : i \in \mathcal{N})$. 
Exchange of Connectivity Services

A connected node relays one other node among those one-hop away.

- Instantaneous relay configuration \( R(t) = (R_{ij}(t) \in \{0, 1\} : i, j \in \mathcal{N}) \).

Goal of the service: connect unconnected nodes.

Benefit of each node: amount of relay he receives.
Connectivity among the nodes is fixed and bidirectional.

Connectivity of a mobile with infrastructure, thus opportunity for relaying, varies randomly.

Each node $i$ generates connectivity tokens with rate $D_i$ that she distributes to her neighbors.

Each node $i$ accumulates connectivity tokens, $d_{ji}(t)$ is the number of tokens received from node $j$ by node $i$ till time $t$.

Cumulative number of tokens received by $i$ is $r_i(t) = \sum_j d_{ji}(t)$.

The objective of each node is to maximize $r_i(t)$. 
Long Term Average Regime

\[ \lim_{t \to \infty} \frac{d_{ij}(t)}{t} = d_{ij} \]

\[ D_i = \sum_j d_{ij} \]

\[ r_i = \sum_j d_{ji} \]
Long Term Average Regime

- An undirected connected graph \( G = (\mathcal{N}, \mathcal{E}) \).

\[
D_i \quad d_{ij} \quad \mathcal{N}_i \quad d_{ji} \quad r_i
\]

- Set of exchange configurations:

\[
\mathbb{D} = \{ \mathbf{d} = (d_{ij})_{(i,j) \in \mathcal{E}} : d_{ij} \geq 0, \sum_{j \in \mathcal{N}_i} d_{ij} = D_i \}\]

- Set of feasible received resource vectors:

\[
\mathbb{R} = \{ \mathbf{r} = (r_i)_{i \in \mathcal{N}} : r_i = \sum_{j \in \mathcal{N}_i} d_{ji}, i \in \mathcal{N}, \mathbf{d} \in \mathbb{D} \},
\]

- Exchange ratio vector:

\[
\rho_i = \frac{r_i}{D_i}, \quad \mathbf{\rho} = (\rho_i, i \in \mathcal{N})
\]

**What are sensible exchange configurations and received resource vectors?**
Which is a *fair* exchange configuration?

- Ideal exchange: \( r_i = D_i, \ \forall \ i \in \mathcal{N}, \) i.e., \( \rho_i = 1 \)
- Else: balance the exchange ratios as much as possible.

Lexicographically optimal (Max-min fair) vector of exchange ratios \( \rho. \)

- If \( x \preceq_{\text{lex}} y, \ \forall \ y, \) then \( x^* \) is lex-optimal, where \( x, y \in \mathbb{R}^N. \)

Is there a lex-optimal vector of exchange ratios \( \rho^* \succeq \rho? \)

- Where we defined \( \rho_i = r_i/D_i. \)

Which are the exchange configurations \( d^* \) that yield \( \rho^*? \)
Coalitional Framework

- Assume that subsets of nodes can jointly decide to exclude others.

- NTU Coalitional Service Exchange game:
  - Played over the graph $G = (\mathcal{N}, \mathcal{E})$, by $\mathcal{N}$ players.
  - Each node $i$ has strategy $d_i = (d_{ij} : j \in \mathcal{N}_i, \sum_j d_{ij} = D_i)$, and utility $\sum_j d_{ji}$.

- (Strong) Stability Definition:
  - An allocation $d$ (and the resource vector $r$) is called strongly stable if $\forall S \subseteq \mathcal{N}$, there is no allocation $\mathbf{d}_S$ on the induced subgraph $G_S = (S, \mathcal{E}_S)$, such that $\hat{r}_i \geq r_i \ \forall i \in S$, and $\hat{r}_j > r_j$ for at least one node $j \in S$.

- Are there strongly stable exchanges? Can we compute them?
Market Framework (Arrow-Debrau)

- $\lambda_i$: selling price of node $i$ in $/\text{unit resource}$.
- $d_{ij}$: amount of node $i$ resource purchased by node $j$.
- Exchange $d_{ij}$, $i = 1, 2, \ldots, N$, $(i, j) \in E$.
- An exchange is **feasible** if each node $i$ can afford his acquisitions by his sales:
  \[
  \lambda_i \sum_j d_{ij} \geq \sum_j \lambda_j d_{ji}
  \]
- An exchange is **stable** if each node $i$ buys only from his cheapest neighbors:
  \[
  d_{ji} > 0 \Rightarrow \lambda_j = \min_{l:(l,i) \in E} \{ \lambda_l \}
  \]
- An exchange is **efficient** if each node $i$ sells all his endowment:
  \[
  \sum_j d_{ji} = D_i
  \]
- Are there set of prices $(\lambda_1, \lambda_2, \ldots, \lambda_N)$ such that there is an exchange that is simultaneously **feasible**, **stable** and **efficient**?
Results

- There is a unique lex-optimal exchange ratio vector $\rho^*$. 

- There is a set of prices $(\lambda_1^*, \ldots, \lambda_N^*)$, unique up to scaling, for which there are exchanges feasible, stable and efficient.

- The two are related as $\lambda_i^* = \sqrt{\rho_i^*}$.

- All exchanges that are feasible, stable and efficient, are strongly stable with respect to coalitions.

- The opposite does not hold: there are exchanges stable with respect to coalitions that deviate from the lex optimal exchange ratio vector.
Characterization of the lex-optimal vector

- For a graph $G = (\mathcal{N}, \mathcal{E})$, endowments $\{D_i\}$, and $\forall \mathbf{r} \in \mathbb{R}$ define:
  - The different values (levels) of the exchange ratios: $l_1 < l_2 < \ldots < l_K$
  - The level index $k(i)$ of each node $i$: $l_{k(i)} = \rho_i$
  - The level set $\mathcal{L}_m = \{i \in \mathcal{N} : k(i) = m\}$, $m = 1, \ldots, K$.
  - Node subsets:
    - $\mathcal{Q}_1 = \mathcal{N}$, and $\mathcal{Q}_k = \mathcal{N} - \bigcup_{m=1}^{k-1} (\mathcal{L}_m \cup \mathcal{L}_{K-m+1})$, $2 \leq k \leq \lceil K/2 \rceil$.
    - Subgraph $G_{\mathcal{Q}_k} = (\mathcal{Q}_k, \mathcal{E}_{\mathcal{Q}_k})$
  - $\mathcal{N}(S)$: neighbors of nodes in set $S$, which do not belong themselves in $S$. 

![Graph example](image-url)
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- The level index \( k(i) \) of each node \( i \): \( k(i) = p_i \).
- The level set \( \mathcal{L}_m = \{ i \in \mathcal{N} : k(i) = m \} \), \( m = 1, \ldots, K \).
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- The different values (levels) of the exchange ratios: \( l_1 < l_2 < \ldots < l_K \)
- The level index \( k(i) \) of each node \( i \): \( l_{k(i)} = \rho_i \).
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Properties of $\rho^*$

Theorem: If an allocation $d^*$ is lex-optimal, then the following properties hold:

1. $\mathcal{L}^*_k$ is an independent set in graph $G_{Q_k}$, for $k = 1, \ldots, \left\lfloor \frac{K}{2} \right\rfloor$.
   - E.g., nodes in $\mathcal{L}^*_1$ are independent in $G_{Q_1} = G$, $\mathcal{L}^*_2$ are independent in $G_{Q_2}$, etc.

2. $\mathcal{L}^*_{K-k+1} = \mathcal{N}_{Q_k}(\mathcal{L}^*_k)$, for $k = 1, \ldots, \left\lfloor \frac{K}{2} \right\rfloor$.
   - E.g., (assume $K = 7$) $\mathcal{L}^*_7 = \mathcal{N}_{Q_1}(\mathcal{L}^*_1)$, $\mathcal{L}^*_6 = \mathcal{N}_{Q_2}(\mathcal{L}^*_2)$, etc.

3. $l^*_k \cdot l^*_{K-k+1} = 1$, for $k = 1, \ldots, \left\lfloor K/2 \right\rfloor$.
   - E.g., $l_1 \cdot l_7 = 1$, $l_2 \cdot l_6 = 1$, etc.

4. $\sum_{i \in \mathcal{L}^*_k} r^*_i = \sum_{i \in \mathcal{L}^*_{K-k+1}} D_i$, for $k = 1, \ldots, \left\lfloor \frac{K}{2} \right\rfloor$.
   - E.g., $\sum_{i \in \mathcal{L}^*_1} r^*_i = \sum_{i \in \mathcal{L}^*_7} D_i$, $\sum_{i \in \mathcal{L}^*_2} r^*_i = \sum_{i \in \mathcal{L}^*_6} D_i$

Note that when $K$ is odd, then $l_k = 1$ for $k = \left\lfloor K/2 \right\rfloor + 1$. 
Properties of $\rho^*$

- There is a unique $\rho^*$ and one or more $d^* \in \mathbb{D}$, with properties:
  - Nodes are partitioned in distinct exchange ratio sets $L_1, L_2, \ldots, L_7$.
  - $K = 7$ depends on $G$ and $D_i$, $i = 1, \ldots, N$.
  - $L_7$ nodes work only with $L_1$ nodes, and so on.
  - It holds: $l_1 \cdot l_7 = l_2 \cdot l_6 = \ldots = 1$.

- **Theorem**: If an exchange policy satisfies the above properties, then it is lex-optimal.
Examples

- Tandem networks with 2 and 3 nodes, having resources $D = 1$.

  ![Tandem network diagram](image)

- Triangle network with identical nodes.

  ![Triangle network diagram](image)

- In the two tandems there is a unique exchange configuration for the lex-optimal ratios; while in the triangle all configurations for $0 \leq a \leq 1$ give the lex-optimal ratio.
Examples

- Binary tree network with 3 levels; identical nodes, $D = 1$. 

![Diagram of a binary tree network with 3 levels and identical nodes, showing the value $D = 1$.]
Examples

- Impact of endowments.
- Complete graphs of 6 nodes; slightly different endowments.
- Left: $K = 1$, Right: $K = 2$.
- Complete graphs have at most $K = 2$:
  - Whenever the maximum endowment exceeds the sum of the rest.
Examples

- \( r_1^* = 26, r_2^* = 20, r_3^* = 39.74, r_4^* = 42.78, r_5^* = 93.49, r_6^* = 14.97, r_7^* = 30.38, r_8^* = 20.96, r_9^* = 30.38, r_{10}^* = 4.28, r_{11}^* = 160, r_{12}^* = 6.25, \) and \( r_{13}^* = 33.75. \)

- \( K^* = 6 \) levels: 0.25, 0.4278, 0.7692, 2.3373, 1.3, 4.

- Level sets: \( \mathcal{L}_1^* = \{12, 13\}, \mathcal{L}_2^* = \{4, 6, 8, 10\}, \mathcal{L}_3^* = \{2\}, \mathcal{L}_4^* = \{1\}, \mathcal{L}_5^* = \{3, 5, 7, 9\}, \) and \( \mathcal{L}_6^* = \{11\}. \)
Examples

- Impact of graph.
- $K^* = 4$ levels: 0.45, 0.77, 1.3, 2.22
- Level sets: $\mathcal{L}_1^* = \{4, 6, 8, 10, 13\}$, $\mathcal{L}_2^* = \{2\}$, $\mathcal{L}_3^* = \{1\}$, $\mathcal{L}_4^* = \{3, 5, 7, 9, 11, 12\}$.
- What has changed?
  - $K = 4$ instead of $K = 6$.
  - Node 12 went from lowest to highest level, while 13 stayed in the lowest!
  - Node 6 changed relative ranking, although he is not connected, nor has a common neighbor with nodes 12, 13.
Scaling the lex-optimal vector

- Scaling the tandem network; assume all node endowments equal to 1.

\[
\begin{align*}
\text{r=1} & \quad 1 & \quad \text{r=1} \\
1 & & \\
0.5 & & 0.5 \\
\text{r=0.5} & \quad 1 & \quad \text{r=2} & \quad 1 & \quad \text{r=0.5} \\
0.5 & & & & \\
\text{r=1} & \quad 1 & \quad 1 & \quad \text{r=1} \\
1 & & & & \\
2/3 & \quad \text{r=1.5} & \quad 0.5 & \quad 1/3 & \quad \text{r=1.5} & \quad 1 & \quad \text{r=2/3} \\
\frac{n}{n+1} & \quad \frac{n+1}{n(n+1)} & \quad \frac{1}{n} & \quad \frac{n}{n+1} & \quad \frac{(n-1)(n+1)}{n} & \quad \frac{n+1}{n} & \quad \frac{2}{n} & \quad \frac{n}{n+1} & \quad \frac{(n-2)(n+1)}{n} \\
1 & & & & & & & & \\
\end{align*}
\]
Binary tree network with 3 levels; identical nodes, $D = 1$. 
Scaling

- Scaling the tree network.
Scaling

- Odd level nodes obtain resource 2; even level nodes resource 0.5.
Dynamic Network Operation

- Node $i$ creates "service token" (e.g., relay opportunity) with a certain probability based on the average generation rate $D_i$.

- Node $i$ keeps track of the number of tokens $d_{ij}(t)$ that were given to $j$, and the number of tokens $d_{ji}(t)$ received by $j$ until time $t$.

- Proportional allocation policy by each node $i$ in slot $t$:
  - Announces to its neighbors its current aggregate exchange ratio:
    \[ \rho_i(t) = \frac{r_i(t)}{D_i(t)}, \]
    where $r_i(t) = \sum_{\tau=1}^{t} \sum_j d_{ji}(\tau)$, and $D_i(t) = \sum_j d_{ji}(t)$.
  - Allocates its generated tokens to the neighbor $j$ that has the smallest ratio $r_j(t)$.

- The ratio converges to the lex-optimal point:
  \[ \lim_{t \to \infty} \rho_i(t) = \rho_i^* \]

---

Dynamic Network Operation

- Assume each node $i$ only keeps track of the tokens she gives and receives from each neighbor $j$, $d_{ij}(t)$, $d_{ji}(t)$.

- An alternative policy is to rely on bilateral interactions, where $i$ allocates its resource to $j$ such that:
  \[
  j^* = \arg \min_{j: (i,j) \in \mathcal{E}} \frac{d_{ij}(t)}{d_{ji}(t)},
  \]

- We observe numerically that under this policy we also have $\rho_i(t) \to \rho^*$. 

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What if the connections are not bidirectional?

- If the links are not bidirectional, obvious options of collaboration will not be activated under bilateral allocation strategies.

- Mechanisms that rely on some form of currency that circulates are needed so that cycles with more than two nodes are activated.